1 Blackbody Radiation

Assuming the energy of radiation to be continuously variable show that $\langle E\rangle = kT$ and obtain the Rayleigh-Jeans radiation formula. Assuming the energy of radiation to be quantized, also show that $\langle E\rangle = hv/[e^{hv/kT} - 1]$ and obtain the Planck Radiation formula.

1.1 Solution

We seek to prove that $\langle E\rangle = kT$. The average of any function is given as

$$\langle E\rangle = \frac{\int_0^\infty EP(E)\,dE}{\int_0^\infty P(E)\,dE}$$

As $P(E)$ is usually normalized, $\int_0^\infty P(E)\,dE = 1$, but we may demonstrate this rigorously knowing that $P(E) = e^{-E/kT}/kT$. Why is $P(E)$ defined to be $e^{-E/kT}/kT$? I will explain this rigorously in the derivation section.

$$\int P(E)\,dE = \int_0^\infty \frac{e^{-E/kT}}{kT}\,dE = \frac{1}{kT}\int_0^\infty e^{-E/kT}\,dE = -\frac{kT}{kT}e^{-E/kT}\Big|_{E=0}^{E=\infty} = -(e^{-\infty/kT} - 1) = 1$$

We now seek to evaluate $\int_0^\infty EP(E)\,dE$, which may be expanded as

$$\int_0^\infty Ee^{-E/kT}/kT\,dE = \frac{1}{kT}\int_0^\infty Ee^{-E/kT}\,dE$$

We execute integration by parts with $u = E, du = dE$ and $dv = e^{-E/kT}dE$, where $v = -kTe^{-E/kT}$.

$$\frac{1}{kT}\int_0^\infty Ee^{-E/kT}\,dE = \frac{1}{kT}\left[(E)(-kTe^{-E/kT})\Big|_{E=0}^{E=\infty} - \int_0^\infty -kTe^{-E/kT}\,dE\right] = \frac{1}{kT}\int_0^\infty kTe^{-E/kT}\,dE$$

$$\frac{1}{kT}\int_0^\infty Ee^{-E/kT}\,dE = \int_0^\infty e^{-E/kT}\,dE = kTe^{-E/kT}\Big|_{E=0}^{E=\infty} = kT(e^{-\infty/kT} - 1) = \boxed{kT}$$

To obtain the Rayleigh-Jeans Law for Blackbody Radiation, our strategy will be to find the number of electromagnetic harmonics that can fit inside a blackbody – a Jeans Cube, for simplicity.

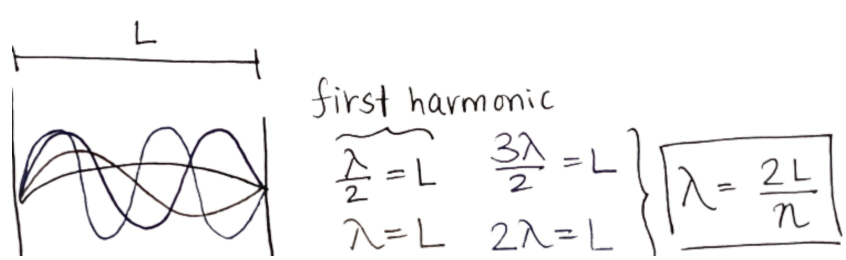

Figure 1: Wavelength of nth Harmonic

Armed with the wavelength of the nth Harmonic, we may express the electromagnetic wave as $A\sin(k_x x)\sin(k_y y)\sin(k_z z)$, where the wavenumbers may be expressed as

$$k_x = \frac{2\pi}{\lambda_x} = \frac{2\pi}{\frac{2L}{n}} = \frac{n\pi}{L}, k_y = \frac{2\pi}{\lambda_y} = \frac{m\pi}{L}, k_x = \frac{2\pi}{\lambda_z} = \frac{l\pi}{L}$$

We thus have the total magnitude of the wavenumber k as

$$k^2 = k_x^2 + k_y^2 + k_z^2 = \left(\frac{n\pi}{L}\right)^2 + \left(\frac{m\pi}{L}\right)^2 + \left(\frac{l\pi}{L}\right)^2 = \frac{\pi^2}{L^2}\underbrace{(n^2+m^2+l^2)}_{p^2}$$

$$k = \frac{\pi}{L}p \to dk = \frac{\pi}{L}dp$$

To find the number of waves that can fit in the momentum space, we consider a spherical volume

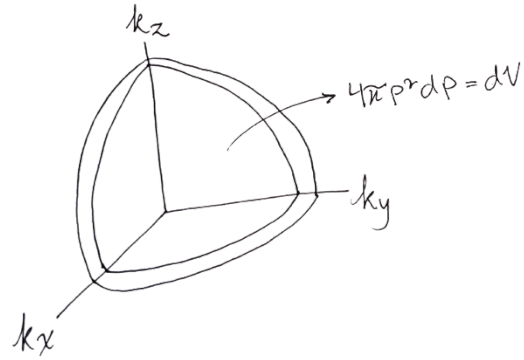

Figure 2: Sphere in Momentum Space

We consider only the part of the spherical volume which is in the first octant, to avoid $k < 0$, which results in a factor of $\frac{1}{8}$. We know that $dp = \frac{L}{\pi}dk$ from above, which gives

$$dV = \frac{1}{8}4\pi p^2 dp = \frac{1}{8}4\pi\left(\frac{Lk}{\pi}\right)^2\left(\frac{L}{\pi}dk\right) = \frac{4\pi L^3k^2}{8\pi^3}dk = \frac{L^3k^2}{2\pi^2}dk = \frac{Vk^2}{2\pi^2}dk$$

L^3 is a volume, so I've replaced it with V. Remember, the whole name of the game is to come back to the frequency of the electromagnetic wave (or synonomously, wavelength). To do so, we recall

$$\frac{\omega}{c} = k \to k = \frac{2\pi f}{c} \to dk = \frac{2\pi}{c}df$$

$$d\rho = \frac{k^2}{2\pi^2}dk \to d\rho = \frac{\left(\frac{2\pi f}{c}\right)^2}{2\pi^2}\frac{2\pi}{c}df = \frac{\frac{4\pi^2 f^2}{c^2}}{\pi c}df = \frac{4\pi^2 f^2}{\pi c^3}df$$

Owing to the two possible polarizations of electromagnetic waves, of which all other polarizations are linear combinations, we multiply this density by a factor of 2 to obtain

$$\boxed{d\rho = \frac{8\pi f^2}{c^3}df \to \rho = \frac{8\pi f^2}{c^3}kT}$$

We have finally derived the Rayleigh-Jeans radiation radiation formula. We now seek to derive that if the energy of radiation is quantized, the average energy is

$$\langle E \rangle = \frac{hf}{e^{hf/kT} - 1}$$

The derivation will proceed as follows:

1. Let N_i be the number of oscillators in state E_i.
2. The probability goes down as $e^{-\Delta E/kT}$, where ΔE is the excess energy.
3. The number of oscillators N_1 in the first state will be however many were in the second state multiplied by the probability $e^{-hf/kT}$. Repeat until the ith state N_i to get the total number of oscillators N_{tot}
4. What about the energy of each of these oscillators? The ground state has no energy. The first state has energy hf for each of the N_1 oscillators. The second state has energy $2hf$ for each of the N_2 oscillators. Likewise for the ith state.

The diagram below illustrates our strategy of finding E_i and P_i for each energy level.

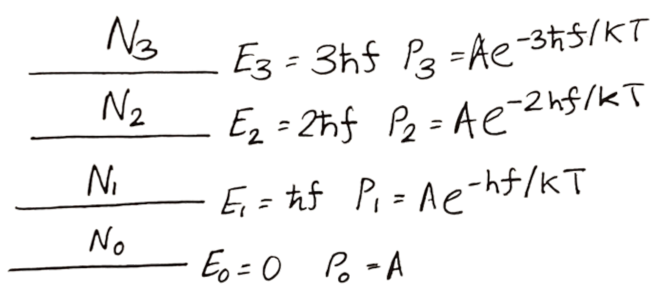

Figure 3: Energy Levels of a Harmonic Oscillator

The energy of the ground state is thus 0. For the first state, we have

$$E_1 = N_1 * hf = N_0 e^{-hf/kT} * hf$$

Likewise for the second state, we have

$$E_2 = N_2 * 2hf = N_0 (e^{-hf/kT})^2 * 2hf$$

As it seems to repeat many times, we denote $x = e^{-hf/kT}$. Our total energy is thus of the form

$$E_{tot} = E_1 + E_2 + E_3 + ... = N_0 e^{-hf/kT} * hf + N_0 (e^{-hf/kT})^2 * 2hf + N_0 (e^{-hf/kT})^3 * 3hf + ..$$

$$E_{tot} = E_1 + E_2 + E_3 + ... = N_0 x * hf + N_0 x^2 * 2hf + N_0 x^3 * 3hf + .. = N_0 hf(x + 2x^2 + 3x^3 + ...)$$

As for the total number of oscillators N_{tot}, we have

$$N_{tot} = N_0 + N_1 + N_2 + N_3 + ... = N_0 + N_0 * e^{-hf/kT} + N_0 * (e^{-hf/kT})^2 + + N_0 * (e^{-hf/kT})^3 + ...$$

Substituting for $x = e^{-hf/kT}$, we have

$$N_{tot} = N_0 + N_0 * x + N_0 * x^2 + +N_0 * x^3 + ... = N_0(1 + x + x^2 + x^3 + ...)$$

The average energy is thus

$$\langle E \rangle = \frac{E_{tot}}{N_{tot}} = \frac{N_0 hf(x + 2x^2 + 3x^3 + ...)}{N_0(1 + x + x^2 + x^3 + ...)}$$

We begin with the infinite summation on the denominator:

$$\begin{aligned} S &= 1 + x + x^2 + x^3 + ... \\ -(Sx &= x + x^2 + x^3 + ...) \end{aligned}$$

Subtracting these two sums clearly gives $S(1-x) = 1 \rightarrow S = \frac{1}{1-x}$. We may execute a similar strategy for the sum on the numerator:

$$\begin{aligned} S &= x + 2x^2 + 3x^3 + ... \\ -(Sx &= x^2 + 2x^3 + ...) \end{aligned}$$

Subtracting the two infinite sums gives $S(1-x) = x(1 + x + x^2 + ...)$. We recognize this sum!

$$S(1-x) = x\frac{1}{1-x} \rightarrow S = \frac{x}{(1-x)^2}$$

Making the necessary substitutions into $\langle E \rangle$, we have

$$\langle E \rangle = \frac{E_{tot}}{N_{tot}} = \frac{N_0 hf \frac{x}{(1-x)^2}}{N_0 \frac{1}{1-x}} = hf\frac{x(1-x)}{(1-x)^2} = \frac{hfx}{1-x} * \frac{\frac{1}{x}}{\frac{1}{x}} = \frac{hf}{x^{-1} - 1} = \boxed{\frac{hf}{e^{hf/kT} - 1}}$$

Multiplied by the number density of waves, Planck's Blackbody Radiation Law is

$$\boxed{\rho = \frac{8\pi f^2}{c^3}\frac{hf}{e^{hf/kT} - 1}}$$

Voíla! Above lies the one-inch equation which would revolutionize our understanding of the universe.

1.2 Derivation

I promised I would explain why quantum mechanics dictates $P(E) = e^{-hf/kT}$. While I cannot supply the full explanation here, the following brief thought experiment from Feynman may illuminate matters a bit. Thought it may seem wholly unrelated, consider the following situation: the atmosphere – but not the Earth's atmosphere. No, a constant-temperature atmosphere. Should the temperature differ between heights, we have a means of achieving thermal equilibrium, by connecting a socket from a higher pressure region below to a lower pressure region above. The question is this: how does the number of atoms n in the atmosphere change as we go up? As we will shortly see, this is nearly analogous to finding the number of oscillators as we ascend the energy levels of a harmonic oscillator. The ideal gas law states that $PV = NkT$ (don't mind the change in constant), with $n = N/V$ being the number of molecules per unit volume. After some basic pressure calculations which may be found in Chapter 40 of Feynman's Lectures, we find that $\frac{dn}{dh} = -\frac{mg}{kT}n$. It is from this differential equation that the probability $e^{-E/kT}$ arises.

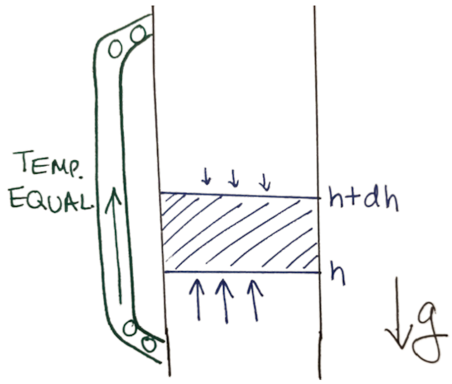

Figure 4: Feynman's Thought Experiment (Ch. 40)

2 Blackbody Radiation #2

The universe is filled with blackbody radiation at 2.7 K left over from the Big Bang. (a) What is the total energy density of this radiation? What is the total energy density with wavelengths between 1 mm and 1.01 mm? Is Rayleigh-Jean's formula a good approximation at these wavelengths? (c) Over what range of frequencies does the Rayleigh-Jean's formula give a result within 10% of Planck blackbody spectrum?

2.1 Solution

Employ the Stefan-Boltzmann Law, we have

$$U = \sigma T^4 = (5.67 * 10^{-8}\text{W/m}^2K^4)(2.7^4K^4) = \boxed{3.01 * 10^{-6}W/m^2}$$

We seek to find the total energy density between $\lambda_1 = 1mm$ and $\lambda_2 = 1.01mm$. Our first order of business will be to convert this into the frequencies $f_1 = \frac{c}{\lambda_1} = 3{*}10^{11}\text{Hz}$ and $f_2 = \frac{c}{\lambda_2} = 2.97{*}10^{11}\text{Hz}$, as that is the input for our Blackbody Radiation formula. Yes, we can convert our original formula, but this strategy is faster. Essentially, all we are doing is finding the area under the blackbody curve. We may as well integrate, but since the domain of integration is so small, we can also take the frequency at $\lambda = 1.005mm$ and simply multiply this by Δf. For the time being, I will simply substitute the known frequencies into the Planck Blackbody Radiation formula and calculate their difference as follows:

$$\rho(f_1 = 3{*}10^{11}\text{Hz}) = \frac{8\pi(3*10^{11}\text{Hz})^2}{(3*10^8\text{m/s})^3}\frac{(6.62*10^{-34}\text{m}^2\text{kg/s})(3*10^{11}\text{Hz})}{e^{(6.62*10^{-34})(3*10^{11})/(1.38*10^{-23})(2.7)}-1} = 8.09{*}10^{-26}\text{Hz}^3\text{kg s}^2\text{/m}$$

$$\rho(f_2 = 2.97{*}10^{11}\text{Hz}) = \frac{8\pi(2.97*10^{11}\text{Hz})^2}{(3*10^8\text{m/s})^3}\frac{(6.62*10^{-34}\text{m}^2\text{kg/s})(2.97*10^{11}\text{Hz})}{e^{(6.62*10^{-34})(2.97*10^{11})/(1.38*10^{-23})(2.7)}-1} = 8.289{*}10^{-26}\text{Hz}^3\text{kg s}^2\text{/m}$$

$$\Delta\rho = \rho(f_2) - \rho(f_1) = (8.289*10^{-26} - 8.09*10^{-26})\text{Hz}^3\text{kg s}^2\text{/m} = \boxed{1.996*10^{-27}\text{Hz}^3\text{kg s}^2\text{/m}}$$

Rayleigh-Jeans is a suitable approximation for low frequencies and high wavelengths. Clearly, at such exceedingly high wavelengths in the millimeter range, Rayleigh-Jeans is a fine approximation. To calculate the range of frequencies over which Rayleigh-Jeans gives a result within 10%

of Planck's blackbody spectrum, I created a python program to numerically compute their difference until it exceeded 10%. Analytically, we may simply calculate when the difference in the Planck and Rayleigh-Jeans energy densities exceeds 10%. The result was that for any wavelength $\lambda > 6675.375375375375$ nm, Rayleigh-Jeans gives a result within the desired threshold of Planck's.

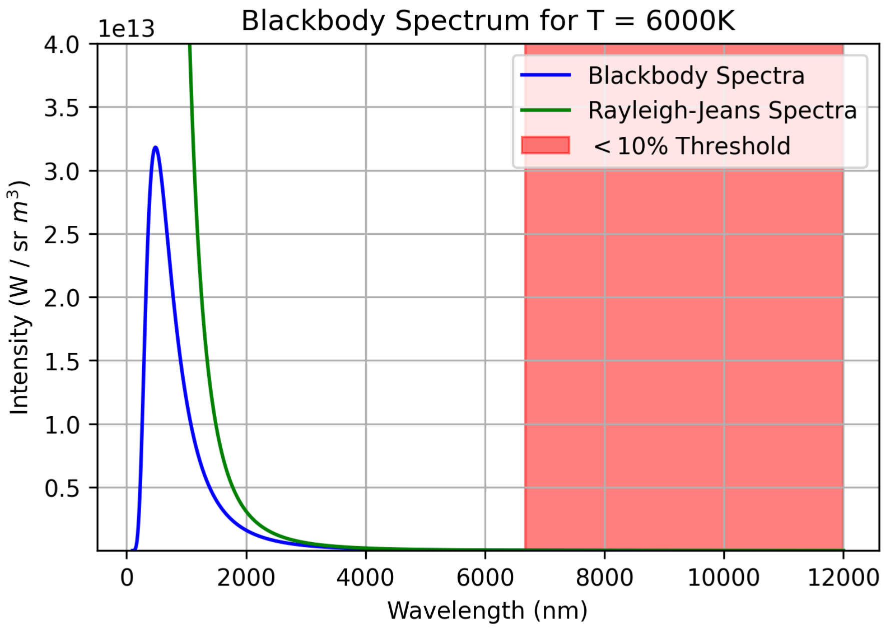

Figure 5: Result of Blackbody Radiation Program

2.2 Derivation

Rayleigh-Jeans is a good approximation in the low frequency regime where $hf << kT$. In fact, Planck's Blackbody Radiation Law approaches Rayleigh-Jean at low frequencies:

$$e^x = \sum_n \frac{x^n}{n!} \rightarrow e^{hf/kT} = 1 + \frac{hf}{kT} + \frac{\left[\frac{hf}{kT}\right]^2}{2!} + \frac{\left[\frac{hf}{kT}\right]^3}{3!} + ...$$

Let's do a second order approximation so that

$$e^{hf/kT} \approx 1 + \frac{hf}{kT} \rightarrow \rho = \frac{8\pi f^2}{c^3}\frac{hf}{e^{hf/kT} - 1} = \frac{8\pi f^2}{c^3}\frac{hf}{\left(1 + \frac{hf}{kT}\right) - 1} = \boxed{\frac{8\pi f^2}{c^3}kT}$$

3 Photoelectric Effect

Suppose light of total intensity 1.0 μW/cm^2 falls on a clean iron sample 1.0 cm^2 in area. Assume that the sample reflects 96% of the light and only 3% of the absorbed energy lies in the violet region of spectrum above the threshold frequency of $1.1 * 10^{15}$ Hz. What effective intensity is available for generating photoelectrons? Assuming that all the photons in the violet region have an effective wavelength of 250 nm, how many electrons will be emitted per second? What will be the magnitude of photoelectric current? Find the work function of the surface in electron-volts, and stopping voltage required for zero current.

3.1 Solution

The effective intensity is 3% of the light which lies in the desired frequency range of the 4% of the light which is absorbed. This translates to $0.03 * 0.04 * 1\mu\text{W/cm}^2 = \boxed{0.0012\mu\text{W/cm}^2}$

How do you get time involved in a question about the photoelectric effect? The key is *Power*. Recall that $P = \frac{W}{t}$. If we are able to find the power on a section of the iron sample and the Kinetic Energy given to the electrons, we are home free. How can we find Power? Recall that

$$P = IA = (0.0012)(1) = 0.0012\mu W$$

I did not convert the area to square meters, since the intensity of the light source is also given in cm^2, so that they will cancel anyway. As for the energy of the photons, we have

$$E = \frac{hc}{\lambda} = \frac{(6.62 * 10^{-34})(3 * 10^8)}{250 * 10^{-9}} = 7.944 * 10^{-19} J$$

But we don't have one photon – we have N photons, with a total energy of $N(7.944 * 10^{-19} J)$. Our goal is to determine the number of electrons ejected per second, $\frac{N}{t}$

$$P = \frac{NE}{t} \rightarrow \frac{N}{t} = \frac{P}{E} = \frac{0.0012\mu W}{7.944 * 10^{-19} J} = \boxed{1.51 * 10^{15}}$$

Finding the Work Function is elementary. All we need to know is the minimum frequency neede to eject these electrons – and we know this! It's $f_0 = 1.1 * 10^{15}$ Hz. For a metal, we have $\phi = hf_0 = (6.62 * 10^{-34})(1.1 * 10^{15}) = \boxed{1 * 10^{-18} J}$ To determine the stopping potential, we begin with the photoelectric equation

$$KE = \phi - E \rightarrow eV = hf_0 - KE \rightarrow V = \frac{hf_0 - KE}{e} = \frac{1 * 10^{-18} - 7.944 * 10^{-19}}{1.6 * 10^{-19}} = \boxed{-4.9 * 10^{38} V}$$

4 Photoelectric Effect #2

The work function of sodium is 2.3 eV. What is the maximum wavelength of light that will cause photoelectrons to be emitted from a sodium surface? What will be the maximum kinetic energy of the photoelectrons if light of wavelength 200 nm shines on a sodium surface? (c) If the power of the 200-nm beam is 5.0 mW, how many photoelectrons will be ejected from the surface in 5 minutes? Assume every incident photon ejects a photoelectron, and that the electrons on the metal surface are not significantly depleted because of photoelectron ejection.

4.1 Solution

We begin by writing the Photoelectric equation for sodium:

$$E = \phi + KE \rightarrow E = 2.3 + \frac{hc}{\lambda}$$

But think about it. If we're using the *maximum* wavelength possible, we're just scraping by, being as lazy as possible. That means the particle will have no Kinetic Energy. We thus have

$$2.3 = \frac{hc}{\lambda} \rightarrow \lambda = \frac{2.3}{hc} = \frac{2.3}{(6.62 * 10^{-34})(3 * 10^8)} = \boxed{8.64 * 10^{-26} m}$$

To find the maximum Kinetic Energy of the photoelectrons when 200 nm light is shone upon the sodium surface, we employ the Photoelectric equation as follows:

$$E_{photon} = \phi_{min} + KE \rightarrow KE = E_{photon} - \phi_{min} = \frac{hc}{\lambda} - 2.3 = \frac{(6.62 * 10^{-34})(3 * 10^8)}{200 * 10^{-9} - 2.3}$$

$$KE_{max} = 9.93 * 10^{-19} J - 3.68 * 10^{-19} J = \boxed{6.25 * 10^{-19} J}$$

To find the number of electrons ejected in 5 minutes, we leverage the same trick we used in the previous photoelectric problem. Namely, we know that power is defined as

$$P = \frac{W}{t} = \frac{NE}{t} \rightarrow N = \frac{Pt}{E} = \frac{(5 * 10^{-3}\text{W}) * (5 * 60\text{s})}{6.25 * 10^{-19}\text{J}} = \boxed{2.4 * 10^{18} \text{ electrons}}$$

5 Compton Scattering

Show that when a photon of energy E is scattered from a free electron at rest, the maximum kinetic energy of the recoiling electron is given by

$$K_{max} = \frac{E^2}{E + m_0 c^2/2}$$

5.1 Solution

I was initially puzzled on how to solve this. I started by using conservation of energy and conservation of momentum, as follows:

$$E_{photon_i} + E_{electron_i} = E_{photon_f} + E_{electron_f}$$

$$hf + m_0 c^2 = hf' + K$$

But I was short of a crucial parameter: what is f'? Aha! Enter Compton Scattering. The Compton Wavelength Shift is given as

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$$

For the electron to gain the maximum kinetic energy, the photon must give off almost all of its initial energy (but – the photon *cannot* be absorbed into the electron! We may show this quite simply using conservation of energy and the total relativistic energy of the electron. This will ultimately result in $p_e = 0$, which implies that $\lambda = \infty$, which is impossible). I thus thought that $\phi = \pi$, which would result in

$$\lambda' - \lambda = \frac{2h}{mc} \to \lambda' = \lambda + \frac{2h}{mc}$$

Solving for K in the conservation of energy equation, we have

$$K = hf' - hf + m_0c^2 \to K = h\left(\frac{c}{\lambda'} - \frac{c}{\lambda}\right) + m_0c^2 = hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right) + m_0c^2$$

$$K = hc\left(\frac{1}{\lambda + \frac{2h}{mc}} - \frac{1}{\lambda}\right) + m_0c^2 = \frac{\lambda}{\lambda(\lambda + \frac{2h}{mc})} - \frac{\lambda + \frac{2h}{mc}}{\lambda(\lambda + \frac{2h}{mc})} = \frac{-\frac{2h}{mc}}{\lambda(\lambda + \frac{2h}{mc})}$$

$$\boxed{KE = -\frac{2h}{\lambda^2 mc + 2h\lambda}}$$

I'm not sure where my derivation went wrong. Obviously, KE cannot be wrong, and it nowhere near matches the correct answer, but I'm not quite sure how to fix my derivation.

5.2 Derivation

Let's derive the Compton Scattering Wavelength shift. The key is to setup the diagram correctly.

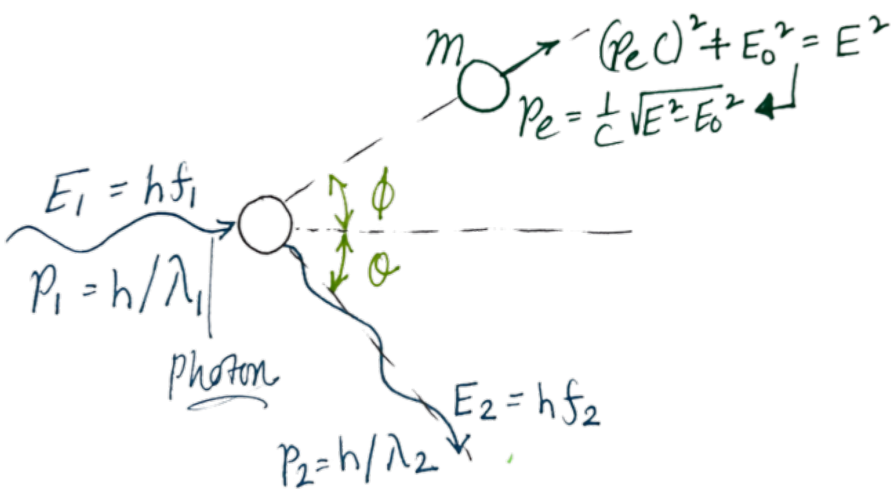

Figure 6: Compton Scattering Diagram

By Conservation of Energy, we have

$$p_1c + E_0 = p_2c + (E_0^2 + p_e^2c^2)$$

Let's move the p_2 term and square both sides to get rid of the square root:

$$E_0^2 + c^2(p_1 - p_2)^2 + 2cE(p_1 - p_2) = E_0^2 + p_e^2c^2$$

Solving for p_e^2 and grouping terms, we have

$$p_e^2 = p_1^2 + p_2^2 - 2p_1p_2 + \frac{2E_0(p_1 - p_2)}{c}$$

We eliminate p_e^2 from both sides to give

$$\frac{E_0(p_1 - p_2)}{c} = p_1p_2(1 - \cos\theta)$$

We now multiply both sides by $\frac{hc}{p_1p_2E_0}$ and substitute $\lambda = \frac{h}{p}$, which gives Compton's Equation!

$$\lambda_2 - \lambda_1 = \frac{hc}{E_0}(1 - \cos\theta) = \frac{hc}{mc^2}(1 - \cos\theta)$$

Finally, this gives us

$$\boxed{\lambda_2 - \lambda_1 = \frac{h}{mc}(1 - \cos\theta)}$$

6 Compton Scattering #2

A photon of initial energy 0.1 MeV undergoes Compton scattering at an angle of 60°. Find (a) energy of the scattered photon; (b) kinetic energy of the electron which was at rest before the collision; and (c) recoil angle of the electron.

6.1 Solution

To find the energy of the scattered photon, we recall that the energy of a photon is inversely proportional to its wavelength λ. We thus must first compute the Compton wavelength shift of the photon using

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\theta)$$

The photon's initial energy of 0.1 MeV $= 1.602 * 10^{-20}J$ tells us that

$$E_{photon_i} = \frac{hc}{\lambda} \to \lambda = \frac{hc}{1.602 * 10^{-20}J} = 1.23970037 * 10^{-5}m$$

All that remains is to substitute the known quantities into our equation to obtain

$$\lambda' = 1.23970037 * 10^{-5} + \frac{h}{mc}(1 - \cos(60)) = 1.2397005 * 10^{-5}\text{m}$$

As expected, the wavelength has increased, and the photon has thus lost energy!

$$E_{photon_f} = \frac{hc}{\lambda'} = \frac{hc}{1.2397005 * 10^{-5}} = \boxed{1.60199984 * 10^{-20}\text{J}}$$

The Kinetic Energy of the electron at rest may be calculated by a simple conservation of energy calculation:

$$E_{photon_i} + E_{electron_i} = E_{photon_f} + E_{electron_f}$$
$$1.602 * 10^{-20} + m_ec^2 = 1.60199984 * 10^{-20} + E_{electron_f}$$
$$\boxed{E_{electron_f} = 1.602 * 10^{-20} + m_ec^2 - 1.60199984 * 10^{-20}}$$

7 X-Ray Scattering

X-rays are produced in an x-ray tube by electrons accelerated through 50 KV. Let K be the kinetic energy of an electron at the end of the acceleration. The electron collides with a target nucleus and then has a kinetic energy of $K_1 = 0.5K$. (a) What is the wavelength of the emitted X-ray? (b) The electron collides with another target nucleus, and then has the energy $K_2 = 0.5K_1$. What is the wavelength of the associated photon?

7.1 Solution

Let's start at the end: the wavelength of a photon is related to its energy by the classic equatoin

$$E = \frac{hc}{\lambda} \to \lambda = \frac{hc}{E}$$

By conservation of energy, we may find the amount of energy left over for kinetic enregy, which we may then substitute into the equation above. We thus have $K_0 = eV = 50keV, K_1 = 0.5K_0 = 25keV$. It is now clear that there is a kinetic energy of $25keV$, which gives a wavelength

$$\lambda = \frac{hc}{E} = \frac{hc}{25keV} = \boxed{49.6m}$$

What about the emitted photon after the second collision? We employ the same approach, but this time with $K_2 = 0.5K_1 = 0.5 * 25keV = 12.5keV$, which gives

$$\lambda_2 = \frac{hc}{12.5 * (1.6 * 10^{-19})} = \boxed{99m}$$

8 Bragg's Law

An X-ray beam of wavelength λ undergoes first order reflection from a crystal when the angle of incidence to a crystal face is 23°, and an X-ray beam of wavelength 97 pm undergoes third order reflection when its angle of incidence to the same face is 60° . Assuming that the two beams reflect from the same family of reflecting planes find the wavelength and inter-planar spacing.

8.1 Solution

This is a simple application of Bragg's Law, which states that

$$n\lambda = 2d\sin(\theta)$$

This is the equation for constructive interference when X-Rays scatter and experience diffraction in crystals. We know that $\theta = 23°$ for $n = 1$, but we do not know λ. On the other hand, we have $\theta = 60$ for $\lambda = 97$ pm and $n = 3$. This gives

$$(1)\lambda = 2d\sin(23)$$

$$(3)(97pm) = 2d\sin(60) \to \boxed{d = 1.68 * 10^{-10}m}$$

Thus, the wavelength for the first-order diffraction is simply

$$\lambda = 2(1.68 * 10^{-10})\sin(23) = \boxed{1.31 * 10^{-10}m}$$